Heat, Fluorescence, and Stimulated-Emission Power Densities and Fractions in Nd:YAG

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Abstract—We present a comprehensive theory describing the CW balance of heat and fluorescence powers in Nd:YAG and discuss the individual and total heat, fluorescence, and stimulated-emission fractions of the absorbed power, with or without stimulated emission, and as a function of the Nd doping density. \(\chi\) values are presented for both CW and pulsed diode pumping. It is shown that heating due to multiphonon emission and concentration quenching alone cannot account for recent CW measurements of the heat fraction, but that upconversion appears to be a negligible effect.

Index Terms—Fractional thermal loading, heat generation, Nd:YAG.

I. INTRODUCTION

The generation of heat in solid-state laser materials is an important effect in high-average-power lasers and in lasers that are pumped at high-intensity in end-pumped and side-pumped configurations. The ability to accurately predict the amount of heat generated in a laser material is particularly important in determining the size of laser material needed to produce a pre-determined amount of output power, in determining what the laser efficiency will be, and in sizing the cooling loop needed to remove the heat generated in a lasing element. Scaling laws often used for high-average-power performance predictions [1]–[4] are only accurate if the heat deposition in the lasing element is known. The evaluation of thermooptic effects in rod and slab laser elements is critically dependent upon a knowledge of the heat fraction present. Until recently, the only available measurements of heating in Nd-based laser materials used flashlamp pumping [5], [6]. More recently, a number of publications have dealt with certain aspects of heat generation in diode-pumped Nd:YAG, Yb:YAG, and other materials [7]–[10], however, no comprehensive treatment is available that allows the laser designer to calculate the heating generated under a wide variety of conditions. In this paper, we discuss and present a model describing the heat-generating effects in diode-pumped Nd:YAG, the laser material that is most often used in applications and whose properties are more well known than any other laser material. The model discussed here can be a starting point for the description of heat generation in other laser materials. Using a power balance approach, and assuming CW pumping from a monochromatic diode source, we present explicit expressions describing the power density attributable to heat, fluorescence, and stimulated emission. Fractions (of the absorbed power density) describing the heating, fluorescence, and stimulated-emission channels are then presented, with stimulated emission present or not, and as a function of the Nd doping density. It is shown that under all conditions the fractional sum is unity. We also present \(\chi\) values defined for both CW and pulsed pumping.

Calculated fractional heat and \(\chi\) values are then compared to recent experimental results. It is shown that the calculated fractional thermal loading is significantly less than that measured experimentally and that in agreement with previous results [7] a less than unity pumping efficiency that varies with doping density can explain the difference. We also calculate the variation in thermal loading as the stimulated emission ranges from zero to strong saturation. Based on the results presented in this paper, it seems likely that upconversion is not a significant effect in CW Nd:YAG lasers.

II. MODEL DESCRIPTION

We begin by showing in Fig. 1 the simplified energy level diagram for Nd:YAG. Energy level values shown are wavenumbers in cm\(^{-1}\). We assume that monochromatic pumping using a semiconductor diode laser or array takes place, at a wavelength of 808 nm, from the \(4F_{9/2}\) ground state to the \(4F_{7/2}\) pump level which is given the energy (wavenumber) assignment \(\varphi_p = 12.376\) cm\(^{-1}\) [11]. Intramanifold relaxation is ignored. Relaxation from the pump level to the \(4F_{3/2}\) metastable level takes place via the emission of phonons; a potential nonradiative pathway from the pump level back to the ground-state is shown in Fig. 1 and will be discussed later in this paper. The \(4F_{3/2}\) level is assigned the \(R_2\) value of Kaminskii [11], since the difference between the \(R_2\) and \(R_1\) levels is only 84 cm\(^{-1}\). Where possible, we have used the “center-of-gravity” or branching-ratio weighted mean values for the energy of the \(4I_J\) levels to which fluorescence from the \(4F_{3/2}\) level terminates. For the manifold transitions \(4F_{3/2}^{-4I_{11/2}}\) and \(4F_{3/2}^{-4I_{13/2}}\), Kaminskii [11] gives a complete listing of branching ratios; we use the weighted average defined by the equation

\[
\varphi_{m,J} = \frac{\sum_i \beta_{m,J}^{i} \varphi_{i}^{i}}{\sum_i \beta_{m,J}^{i}} = \frac{\sum_i \beta_{m,J}^{i} \varphi_{i}^{i}}{\beta_{m,J}}.
\]

Here \(m\) refers to the metastable state, \(\varphi_{i}\) is the wavenumber to the \(i\)th level of the lower \(J\)th manifold, the sum is over all \(i\) levels of the \(J\)th manifold, \(\beta_{m,J}^{i}\) is the branching ratio to the \(i\)th level of the \(J\)th manifold, and \(\beta_{m,J}\) is the total branching ratio from the metastable state to the \(J\) manifold.
Fig. 1. Energy levels and assignments of Nd:YAG. Thin vertical lines denote radiative transitions and bold vertical lines multiphonon transitions.

The values of $\beta_{m,J}$ are well known, and we use those of [11]: $\beta_{m,9/2} = 0.3$, $\beta_{m,11/2} = 0.5598$, $\beta_{m,13/2} = 0.1377$, and $\beta_{m,15/2} = 0.0025$. These values are close to those quoted in [12] and [13]. Evaluating (1) for the $^4F_{3/2}$$-^4I_{11/2}$ and $^4F_{3/2}$$-^4I_{13/2}$ transitions gives the values $\gamma_{11/2} = 2211 \text{ cm}^{-1}$ and $\gamma_{13/2} = 4260 \text{ cm}^{-1}$ shown in Fig. 1. Since individual branching ratios from $^4F_{3/2}$ to the $^4I_{9/2}$ ground-state manifold are not available, and to simplify the model, we take $\gamma_{9/2} = 0$ and ignore any intramanifold relaxation. The energy level assignment of the $^4I_{15/2}$ state is just the average of the energy level assignments of Kaminskii [11] for that manifold, since the individual branching ratios are not known. This approximation will not affect our results since the total branching ratio $\beta_{m,15/2} = 0.0025$ for that manifold is very small.

Relaxation from the lower $J$ manifolds to the ground-state manifold are assumed to take place via multiphonon emission. In Fig. 1, we also show a possible nonradiative relaxation path from the metastable level to the ground state.

The starting point for our model is the rate equation describing the metastable ion density $n_m(\rho)$ evolution with time $t$, which we write as a function of the Nd$^{3+}$ ion density $\rho$:

$$\frac{dn_m(\rho)}{dt} = n_\text{exc}(t) - k_R n_m - k_Q n_m - k_QE n_m,$$

$$- k_L (n_m - n_T) - \gamma(\rho)n_m^2$$

(2)

where $R_\text{c} = R_\text{c}(t)$ is the excitation density and can be time-dependent, $k_R$ is the metastable radiative rate constant, $k_Q$ the concentration-quenching variable which is dependent upon the Nd ion density, $k_QE$ the rate constant associated with a nonunity metastable radiative quantum efficiency, and $k_L$ is the rate constant due to stimulated emission. Note that we have ignored the metastable and terminal level degeneracies since they are equal and that since the measured terminal-level lifetime has been measured to be very short [16], [17] we take the terminal-level ion density $n_T = 0$. The sixth term on the right-hand side of (2) represents the rate of loss of ions due to upconversion and has a rate variable $\gamma$ that is dependent on the Nd ion density, and an $n_m^2$ dependence. The quantity $n_\text{exc}(\rho)$ multiplying $R_\text{c}$ will be used to account for nonunity transfer efficiency (or nonradiative quantum efficiency) from the pump to the metastable level, or dead sites [7], [13], and in general is a function of the ion density $\rho$. It has been conjectured [7] that the perceived drop in Nd:YAG $^4F_{3/2}$ quantum efficiency as Nd doping is increased is due to “dead sites” that short-circuit or bypass the metastable state. Later in this paper we will examine this possibility in more detail. Equation (2) will be solved in the steady-state limit later in this section and for constant peak power pulsed pumping in Section V. The various terms in (2) will now be discussed.

1) Excitation Density: The excitation density $R_\text{c}(z)$, or the number of excited ions/volume-time, can be shown to be equal to:

$$R_\text{c}(r,z) = \frac{\lambda_\text{p} \lambda_\text{p} \lambda_\text{p}}{\hbar c} I(r,z)$$

(3)

where $\lambda_\text{p}$ is the pump wavelength, $\alpha_\text{p}$ the pump absorption coefficient, $\hbar$ and $c$ Planck’s constant and the speed of light, respectively, and $I(r,z)$ the pump intensity at any radial location $r$ or axial location $z$. In the remainder of this paper, we consider only small volumes over which $I(r,z)$ and $R_\text{c}(r,z)$ are essentially constant, suppress the $r$ and $z$ dependencies, and take $R_\text{c}$ as a constant.
2) Radiative Decay: We describe the rate of metastable radiative decay, \( k_R = 1/\tau_R \), for zero Nd ion density, by using a single radiative lifetime \( \tau_R \) to describe it, and with a value \( \tau_R = 260 \times 10^{-6} \) s or close to the Judd–Ofelt derived lifetime [12]. Direct measurements of the radiative lifetime in low Nd-doped samples of Nd:YAG by a number of researchers have yielded values of 260 \( \mu \)s [19], 272 \( \mu \)s [13], and 260 \( \mu \)s [20].

3) Concentration Quenching: To describe concentration quenching, we have digitized three sets of experimental fluorescence lifetime data [11], [18], [19] \( \tau \) as a function of the Nd ion doping density \( \rho \) expressed in at.\%. The combined data sets are shown in Fig. 3. It can be seen that the three sets of data are substantially different. The data of Kaminskii [11] at 77 and 300 K were digitized directly from [11, Fig. 6.3] and indicate longer lifetimes than either of the other data sets. At this point it is worth mentioning that for Nd doping densities below about 0.4 at.% [19] to 0.7 at.% [21], the fluorescence decay has been found to be described by a single exponential with a well-defined lifetime. For larger doping densities, the decay is nonlinear and consists of a fast initial decay \( \tau_1 \) followed by a longer decay time constant \( \tau_2 \) [19], [21]–[23]. The data of [11] are believed to be \( \tau_2 \) as a function of the doping density; no discussion as to the interpretation of [11, Fig. 6.3] is offered in [11]. The data of [18, Fig. 3] provide the most rapid falloff of fluorescence lifetime with ion density and represent the \( 1/e \) fluorescence lifetime, thus underestimating the true effective lifetime for the larger doping densities. The relative quantum efficiency data from [19, Fig. 5], as a function of doping density, were used to obtain the effective lifetime \( \tau_e \) from the definition

\[
\eta_R = \frac{\tau_e}{\tau_R}
\]

of the quantum efficiency \( \eta_R \). These data, also plotted in Fig. 3, are believed to be the most accurate since they represent the true effective lifetime \( \tau_e \) where \( \tau_e \) is defined by the relationship

\[
\tau_e = \int \frac{I(t)}{I_0} \, dt.
\]

The fluorescence intensity \( I(t) \) is given by

\[
I(t) = I_0 \exp \left( -\frac{t}{\tau_R} + \Pi(t) \right)
\]

where \( \Pi(t) \) is a nonlinear function that describes the kinetics of the nonradiative energy transfer from the donor state and has been measured by Lupei et al. [19] for various Nd dopings from 0 to 2.5 at.\%. Also shown in Fig. 3 are published values of \( \tau \), including \( \tau = 217 \) at 1.1 at.\% Nd doping [9] and 230 \( \mu \)s at 1 at.\% Nd doping [3] which agree well with the data of...
indicating that these are also $1/e$ and not true effective lifetimes.

The three sets of data in Fig. 3 have been nonlinear least-square fitted using the function

$$\tau(\rho) = \tau_R \exp(-b\rho^2), \quad (7)$$

For the data of Lupei et al. [19], the constants used in the fit are $\tau_R = 260 \mu s$, $b = 0.0668$ (at.%)$^{-2}$ and $c = 2.67$; these values are used in the remainder of this paper to describe concentration quenching. For the Kaminskii data [11], we find $\tau_R = 258 \mu s$, $b = 0.118$ (at.%)$^{-2}$, and $c = 3.42$.

We have not used a Dexter-type function, in which $\tau$ is inversely proportional to the square of the Nd doping density [24], [25], because over the doping range of interest, 0–3 at.% Nd, (7) provided a far more accurate fit to the experimental data. In Fig. 4, we show three nonlinear least-square fits to the Kaminskii lifetime data [11], including a Dexter function where $\tau$ is described by the equation

$$\tau = \frac{\tau_R}{(1 + b\rho^2)}, \quad (8)$$

with a similar function

$$\tau = \frac{\tau_R}{(1 + b\rho^2)} \quad (9)$$

and (7). The fits of (7)–(9) are represented by the fine dashed, course dashed, and solid lines, respectively. For (8), the constants are $\tau_R = 256 \mu s$ and $b = 0.0742$ (at.%)$^{-2}$, while for (9) they are $\tau_R = 250 \mu s$, $b = 0.0132$ (at.%)$^{-0.99}$, and $c = 3.93$. Visually, it can be seen that (7) and (9) give almost identical fits over the range of interest while (8) results in a fit that is far from optimum. To better quantify the goodness of fit we have also calculated the standard error $e_s$, formally given by

$$e_s = \left[ \frac{1}{N} \sum_{i=1}^{N} (\tau - \bar{\tau})^2 \right]^{1/2}. \quad (10)$$

Here $N$ is the number of experimental points resulting with lifetimes $\tau$ and $\bar{\tau}$ are the lifetimes calculated from the fits of (7)–(9). Calculating (10) for the fits of (7)–(9), we find $e_s = 5.3$, 11.4, and 5.3 $\mu s$, respectively. The standard error is almost identical for (7) and (9), however (8) gives an error over twice as large. $\tau_R$ was also calculated for the data of Lupei et al. and was found to have the value 2.8 $\mu s$ using (7) as the fitting function. None of the three data sets used in Fig. 3 were found to obey a Dexter-type function [8].

The concentration quenching rate $k_Q(\rho)$ is then written in the form used throughout this paper

$$k_Q(\rho) = k_R \left( \exp(b\rho^2) - 1 \right), \quad (11)$$

4) Non-Unity Metastable Quantum Efficiency: The quantum efficiency of the metastable $4f_{3/2}$ state has been estimated or measured by a number of workers [13], [19], [23], [26] and has been found to be close to unity. Mechanisms leading to a decrease in radiative quantum efficiency have not yet been identified, although a mechanism that bypasses the metastable level entirely has been identified, resulting in a smaller “effective” quantum efficiency [13]. Nevertheless, to account for a possible as yet unidentified mechanism that reduces the radiative lifetime at zero doping, we have included a quantum efficiency $\eta_QE$ in our model, which in the modeling results that follow, takes the value of 1. In the future, if better measurements of radiative lifetime and quantum efficiency are available that indicate a less than unity metastable quantum efficiency, the effect can easily be incorporated in our model. We then take the nonunity quantum efficiency rate $k_{QE}$ to be equal to

$$k_{QE} = k_R(1 - \eta_{QE}), \quad (12)$$

5) Stimulated Emission: To describe stimulated-emission, we use the following to describe the rate constant $k_L$ [3]:

$$k_L = k_R \left( \frac{\varphi}{\varphi_S} \right). \quad (13)$$
ξ = (ω/ωΤ) is the ratio of the photon density or intensity to the saturation density or intensity. For a photon density just equal to the saturation density, the stimulated-emission and fluorescence decay rate constants are equal. Equation (7) can also be used to describe the depletion of the metastable level by the flux due to amplified spontaneous emission (ASE). As we will see later in this paper, stimulated emission from a laser beam or from ASE can reduce the heating produced in Nd:YAG.

6) Upconversion: Upconversion has been identified to be important in some solid-state materials such as Cr:LiSAF [27], [28]. It has been conjectured that the effect is partially responsible for the increase in heat load observed in diode-pumped Nd:YAG amplifiers when stimulated emission is absent [14]. To model this effect, we have incorporated in the last term on the right-hand side of (2) the upconversion rate \( k_{UC}(ρ) \) given by

\[
k_{UC}(ρ) = γ(ρ)n_m(ρ)
\]

where \( γ \) is the upconversion constant and \( k_{UC}(ρ) \) is a function of the Nd ion density \( ρ \). Substituting (11)–(14) in (2), and solving for steady-state conditions \( dn_m/dt = 0 \), yields the following expression for the CW ion density \( n_{cw}^m(ρ, ξ) \) with \( ξ = 0 \):

\[
n_{cw}^m(ρ, ξ) = \frac{η_e(ρ)R_e}{kR(1 + F(ρ) + ξ)}.
\]

The function \( F(ρ) \) can be written

\[
F(ρ) = \left( \exp(hρc^2) - 1 \right) + (1 - η_{QE})
\]

and takes the value 0 if \( ρ = 0 \) and \( η_{QE} = 1 \). If \( γ \) is nonzero, however, we obtain the following expression:

\[
n_{cw}^m(ρ, ξ) = kR(1 + F(ρ) + ξ)2γ(ρ)
\]

\[
\cdot \left( 1 + \frac{4η_eR_e}{kR(1 + F(ρ) + ξ)^2} \right)^{1/2} - 1.
\]

III. CW HEAT, FLUORESCENCE, AND STIMULATED-EMISSION POWER DENSITIES AND FRACTIONS

We now turn our attention to the power densities and fractions associated with each heating effect, with fluorescence, and with stimulated emission.

A. Power Densities

The power density associated with pumping or absorbed power density \( Q_A \) is

\[
Q_A = R_e hω_p.
\]

The power density equations are obtained by multiplying the rate with the photon energy associated with the particular process. Heat power densities used in our model are the power density \( Q_e \) due to a nonunity pump level \(^4\)F\(_{5/2}\) quantum efficiency \( η_e(ρ) \)

\[
Q_e(ρ) = (1 - η_e(ρ))R_e hω_p.
\]

\( Q_{QD}(ρ) \) due to the quantum defect between the pump and metastable levels,

\[
Q_{QD}(ρ) = η_e(ρ)R_e hω_p(ω_p - ω_m)
\]

\( Q_{QF}(ρ) \) due to a nonunity metastable quantum efficiency given by

\[
Q_{QF}(ρ) = kR(1 - η_{QE})n_m(ρ)hω_m
\]

and the heat density \( Q_q \) due to concentration quenching

\[
Q_q(ρ) = kRn_m(ρ)(\exp(hρc^2 - 1))hω_m.
\]

If upconversion is operative, we can calculate the heat power density \( Q_{UC}(ρ) \) associated with that process as

\[
Q_{UC}(ρ) = γ(ρ)n_m(ρ)^2hω_m.
\]

In addition to the above heating contributions, we also have the heat power densities \( Q_{h,J} \) resulting from metastable radiative decay, which can be written as

\[
Q_{h,J}(ρ) = kRn_m(ρ)hβ_m,β_m-J.
\]

The total fluorescence heat power density \( Q_{h,F}(ρ) \) can be obtained by summing up that due to each \( J \) manifold, giving

\[
Q_{h,F}(ρ) = kRn_m(ρ)hCS_J
\]

where the sum \( S_J = \sum_J β_m,β_m-J \), using the “center of gravity” values for \( β_m \) previously discussed, is approximately equal to 1834 cm\(^{-1}\).

Finally, we must also account for the heating associated with stimulated-emission, or \( Q_{h,L}(ρ) \), which can be written

\[
Q_{h,L}(ρ) = kRn_m(ρ)hω-T.
\]

We can sum all heating contributions to obtain the total heat power density \( Q_h(ρ) \) given by

\[
Q_h(ρ) = Q_e(ρ) + Q_{QD}(ρ) + Q_{QF}(ρ) + Q_q(ρ) + Q_{UC}(ρ) + Q_{h,F}(ρ) + Q_{h,L}(ρ).
\]

The fluorescence power densities \( Q_{F,J}(ρ) \) can also be easily obtained, and are given by

\[
Q_{F,J}(ρ) = kRn_m(ρ)hβ_m,β_m-J(ω_m - ω_J).
\]

We can sum (28) over the four \( J \) manifolds to yield the total fluorescence density \( Q_F(ρ) \) given by

\[
Q_F(ρ) = kRn_m(ρ)hCS_J.
\]

If stimulated emission is present, the amount of fluorescence power decreases since from (15) the CW inversion density decreases. We can also find the stimulated emission or laser power density as

\[
Q_L(ρ) = kRn_m(ρ)h(ω_m - ω_T).
\]

With ξ equal to zero, for any \( ρ \) the absorbed power appears as heat and fluorescence only whereas with strongly saturating stimulated emission present, the fluorescence becomes insignificant.
In deriving the previous expressions for the power densities, we have verified that power conservation is obeyed. Thus, as required,

$$Q_A = Q_h(\rho) + Q_F(\rho) + Q_L(\rho).$$  \hspace{1cm} (31)

Finally, in connection with our discussion of CW $\chi$ values in Section IV, we will need the inversion power density $Q_I$, given by

$$Q_I(\rho) = kRn_m(\rho)hc(\omega_m - \omega_T).$$  \hspace{1cm} (32)

B. Fractions

Heat, fluorescence, and lasing fractions $\eta$ are obtained by dividing the aforementioned power densities by the absorbed power density. For the case of no upconversion ($\gamma = 0$), the fractions become particularly simple if (15) is substituted for $\tau_m(\rho)$ in the previous expressions for the power densities. Fractions are summarized below for $\gamma = 0$.

1) Heat Fractions:
   a) Non-unity pump-metastable transfer efficiency:
      $$\eta_{h,PM}(\rho) = \frac{Q_{h,PM}(\rho)}{Q_A} = 1 - \eta_k(\rho)$$  \hspace{1cm} (33)
   b) Pump-metastable quantum defect:
      $$\eta_{h,QD}(\rho) = \frac{Q_{h,QD}(\rho)}{Q_A} = \eta_k(\rho)\left(1 - \frac{\omega_m}{\omega_p}\right)$$  \hspace{1cm} (34)
   c) Non-unity metastable quantum efficiency:
      $$\eta_{h,ME}(\rho) = \frac{Q_{h,ME}(\rho)}{Q_A} = \frac{\eta_k(\rho)(1 - \eta_{DE})}{(F(\rho) + 1 + \xi)} \frac{\omega_m}{\omega_p}$$  \hspace{1cm} (35)
   d) Quenching:
      $$\eta_{h,Q}(\rho) = \frac{\eta_k(\rho)\exp(\omega_p^2) - 1}{(F(\rho) + 1 + \xi)} \frac{\omega_m}{\omega_p}$$  \hspace{1cm} (36)
   e) Upconversion:
      $$\eta_{h,UC}(\rho) = \frac{Q_{h,UC}(\rho)}{Q_A} = 0$$  \hspace{1cm} (37)
   f) Fluorescence:
      $$\eta_{h,F}(\rho) = \frac{Q_{h,F}(\rho)}{Q_A} = \frac{\eta_k(\rho)}{(F(\rho) + 1 + \xi)} \frac{S_J}{\omega_p}$$  \hspace{1cm} (38)
   g) Stimulated emission:
      $$\eta_{h,L}(\rho) = \frac{Q_{h,L}(\rho)}{Q_A} = \frac{\eta_k(\rho)}{(F(\rho) + 1 + \xi)} \frac{\omega_T}{\omega_p}$$  \hspace{1cm} (39)
   h) Total heating fraction:
      $$\eta(\rho) = \left(1 - \eta_k(\rho)\frac{\omega_m}{\omega_p}\right) + \frac{\eta_k(\rho)}{(F(\rho) + 1 + \xi)} \frac{S_J}{\omega_m}$$
      $$\cdot \left(1 + \frac{S_J}{\omega_m} + \frac{\omega_T}{\omega_p}ight)$$  \hspace{1cm} (40)

2) Fluorescence Fraction:

$$\eta_F(\rho) = \frac{Q_F(\rho)}{Q_A} = \frac{\eta_k(\rho)}{(F(\rho) + 1 + \xi)} \frac{\omega_m}{\omega_p} \left(1 - \frac{S_J}{\omega_m}\right).$$  \hspace{1cm} (41)

3) Stimulated-Emission Fraction:

$$\eta_L(\rho) = \frac{Q_L(\rho)}{Q_A} = \frac{\eta_k(\rho)}{(F(\rho) + 1 + \xi)} \frac{\omega_T}{\omega_p} \left(1 - \frac{\omega_T}{\omega_m}\right).$$  \hspace{1cm} (42)

By summing the fractions, it can be verified that $\eta_{h}(\rho) + \eta_{F}(\rho) + \eta_{L}(\rho) = 1$. The inversion fraction, $\eta_0(\rho)$, is given by

$$\eta_0(\rho) = \frac{(\omega_m - \omega_T)}{\omega_p} \frac{\eta_k(\rho)}{(F(\rho) + 1 + \xi)}.$$  \hspace{1cm} (43)

C. Discussion

We first present results in which concentration quenching, quantum defect, and fluorescence are the only heat-generating mechanisms, and stimulated emission is absent. By using (40) with $\eta_k(\rho) = 1$ to calculate the total heat fraction $\eta_{h}(\rho)$ and (41) for the total fluorescence fraction $\eta_{F}(\rho)$, and the concentration quenching fit of (11) with the appropriate fitting constants for the data of Lupei et al. [19], we generated the curves shown in Fig. 5. It can be seen that as the Nd doping increases, the heat fraction increases due to concentration quenching and the fluorescence fraction decreases. For zero doping density, the fluorescence fraction is maximized and takes the value 0.782; the heat fraction is 0.218 and of course the sum of both is equal to 1 for any $\rho$. At 1.0 at.% Nd doping, the heat fraction is calculated to be 0.266. In Fig. 6, we show the heat, fluorescence, and stimulated-emission fractions, assuming that the ratio of stimulated-emission intensity/saturation intensity $\xi = 1/2$. For $\rho = 0$, $\eta_{h}(0) = 0.266$, $\eta_{F}(0) = 0.521$, and $\eta_{L}(0) = 0.251$. 

Fig. 5. Total CW heat (○) and fluorescence (□) fractions as a function of Nd doping, for $K = 0$ and $\xi = 0$. 

Fig. 6. Heat, fluorescence, and stimulated-emission fractions, assuming that the ratio of stimulated-emission intensity/saturation intensity $\xi = 1/2$. For $\rho = 0$, $\eta_{h}(0) = 0.266$, $\eta_{F}(0) = 0.521$, and $\eta_{L}(0) = 0.251$. 


thus the fluorescence fraction is over twice the stimulated-emission fraction. Comparison of Figs. 5 and 6 shows that in the presence of stimulated emission the heat fraction increases for Nd doping levels below about 0.75 at.%, whereas for Nd doping levels above 0.75 at.% the heat fraction decreases. Since most Nd:YAG lasers use Nd doping levels in the range of 1–1.1 at.%, one would expect a decrease in thermal loading if stimulated emission is present.

In Fig. 7, we show the individual heat fractions, again assuming only quantum defect, concentration quenching, and fluorescence heating, and with no stimulated emission present. The quantum defect heating fraction \( \eta_{QD}(\rho) \) (34), concentration-quenching heating fraction \( \eta_{Q}(\rho) \) (36), and the fluorescence heat fraction \( \eta_{F}(\rho) \) (38) are shown as a function of Nd doping. It can be seen that the quantum defect fraction is constant, with the value 0.07. The quenching fraction increases with Nd concentration, and the fluorescence fraction decreases as concentration quenching reduces the inversion density.

It has been shown [7] that quantum defect, concentration quenching, and fluorescence heating are not sufficient to account for measured values of the total heat fraction, and it was proposed that the mechanism for increased heating is “dead Nd sites” that effectively short circuit the metastable level [13]. Measurements and previous observations of laser performance dependence upon Nd:YAG material have led to conjecture [7], [13], [29] that the Nd:YAG materials growth process may be responsible for the significant variations in Nd:YAG laser performance and quantum efficiency. Nd:YAG material with the same Nd doping level can display wide variations in quantum efficiency [7], [13]. Fan [7] treated the possibility that Nd “dead sites” could explain the discrepancy between measured heat fraction values and those calculated using only quantum defect, concentration quenching, and fluorescence as heating mechanisms. For a fixed Nd doping of 1.04 at.%, Fan calculated that about 10% of the Nd ions were dead, or inactive, based on matching theory to the average heat fraction measured (0.39). Here, we consider the same possibility and postulate that the fraction of dead sites increases with Nd doping concentration; we use the following relationship for the pumping efficiency or pump-metastable quantum efficiency:

\[
\eta_p(\rho) = \eta_0(1 - K \rho)
\]  

where \( K \) is a constant and we take the zero-doping value \( \eta_0 = 1 \). This function is used to see what effect a nonunity pump efficiency has on heating in Nd:YAG. We find that for the concentration quenching data we used \( K = 0.155 \) for 1.04 at.% doping, thus the pumping efficiency at that doping concentration is about 0.85. Readers, however, should be cautioned that much work remains to be done concerning the nature of the “dead sites” in Nd:YAG, to determine their cause and dependence on such factors as growth process and impurities, and to ascertain the real functional dependence of pump quantum efficiency on Nd concentration. Equation (44) is simply used here to show how dead Nd sites can account for an increased thermal fraction at 1.04 at.% and how the thermal loading would vary if a linear variation in pump quantum efficiency is assumed.

Fig. 8 shows the effect of adding a linearly varying pump quantum efficiency with \( \eta_0 = 0.155 \). The nonunity pump efficiency heat fraction \( \eta_{QD}(\rho) \) increases linearly with Nd concentration; the quantum defect heat fraction \( \eta_{QD}(\rho) \) is no longer a constant as in Fig. 7, but from (34) linearly decreases as Nd concentration increases. Because less ions reach the metastable level as pump efficiency decreases with increasing Nd concentration, the concentration-quenching heat fraction is less than that of Fig. 7 while the amount of fluorescence also decreases.

In Fig. 9, we show the total heat fraction \( \eta_h(\rho) \) for both \( K = 0 \) and \( K = 0.155 \); it can be seen that the calculated heat load for \( K = 0.155 \) is significantly higher than if a unity pump quantum efficiency is assumed, and that the difference between the two cases is least for lowest Nd dopings. Also
showed in Fig. 9 are the experimental data points measured by Fan [7], McInnes and Richards [9], and Comansky et al. [8]. For a Nd doping of 1.04 at.%, the calculated total heat fraction with \( K = 0.155 \) agrees with the average data point in [7] of \( \eta_h(1.04) = 0.39 \). The data of [8] and [9] for \( \rho = 1.1 \) at.\% fall approximately midway between the two calculated curves for \( K = 0 \) and \( K = 0.155 \). Also shown in Fig. 9 is a curve for \( K = 0.22 \), chosen to match the largest value \( \eta_h(1.04) = 0.44 \), measured in [7], thus the pump quantum efficiency is 0.78. To our knowledge, all of the reported measured values of \( \eta_h \) fall between the curves shown in Fig. 9 for \( K = 0 \) and 0.22. Because, however, the laser designer at present has no way of predicting in advance what the pump quantum efficiency will be for a commercially grown Nd:YAG element, it is prudent to take the total heat fraction for Nd dopings and in the absence of stimulated emission, in the range of 1–1.1 at.\% Nd to be as large as 0.4, and perhaps even as large as 0.45 since data on fractional thermal loading are scarce. From Fig. 9, it appears that a good strategy to decrease heat loading is to use a Nd doping level lower than 1–1.1 at.\%, perhaps to as low as 0.8 at.\%.

It has been reported [14] and discussed [7] that heat loading in Nd:YAG is significantly decreased in the presence of stimulated emission, purportedly because the heating due to a lasing photon is less than that of an average fluorescing photon. Using the theory presented in this paper, we have modeled this effect. Fig. 10 shows the heat fraction for \( K = 0 \) and \( K = 0.22 \), assuming only heating due to quantum defect, concentration quenching, fluorescence, and stimulated emission. For no stimulated emission and \( \rho = 0 \), we have \( \eta_h(0) = 0.218 \), in agreement with [7]. For \( K = 0.155 \), we calculate \( \eta_h = 0.241 \), or just equal to the difference between the pump and laser photon energy, also in agreement with [7]. It can be observed that as stimulated emission becomes stronger the fractional heat loading decreases for \( K > 0.75 \) at.\% while for \( K < 0.75 \) at.\% it increases slightly. At 0.75 at.\% Nd, the drop in the heat load is about 10%.

For low Nd doping concentrations where concentration quenching is minimal, for no stimulated emission the quantum defect heating fraction is 0.07 (see Fig. 7). The remaining heat fraction of 0.148 is due to the “average” terminal fluorescence level, which can be calculated to be 1832 cm\(^{-1}\). The average fluorescence photon then has a wavelength of 1.034 \( \mu \)m. Thus for low Nd dopings and no stimulated emission, a higher average fluorescence photon energy results in a lower heat load. As Nd concentration increases, however, with no stimulated emission, the heat fraction increases as shown previously in Fig. 7, primarily because the rate of ion loss from the metastable level due to concentration quenching becomes comparable to that due to fluorescence. In the presence of stimulated emission, the rate of loss due to stimulated photons competes with concentration quenching and as the stimulated emission becomes strongly saturating is the predominant loss
mechanism. This behavior is demonstrated in Fig. 10. Thus, under conditions of strong laser saturation and for Nd doping greater than about 0.75 at.%, the primary reason for the drop in heat fraction is that stimulated emission predominates over concentration quenching and that stimulated emission produces less heat/event.

In Fig. 11, we show the heat fraction as a function of Nd doping for $K = 0.155$ and $\xi = 0$, 1, $\infty$. For $\rho = 0$, the no-stimulated-emission and strongly saturating fractions are the same as in Fig. 10, and at 1.0 at.% Nd the difference in heat fraction between no stimulated emission and strong saturation is about 6%. In Fig. 12, for a Nd doping of $\rho = 1.1$ at.%, we show the heat fraction as a function of $\xi$, varying between $10^{-2}$ and $10^4$, and for $K = 0$ and 0.155. For $\xi > 10$, the heat fraction has dropped to 0.37 from 0.402 for $K = 0.155$, and to about 0.241 from 0.28 for $K = 0$. These decreases are about 8% and 14%, respectively. In Fig. 13, the same plot is displayed for a Nd doping of $\rho = 0.8$. For $K = 0$ and 0.155, the percentage drop in heat fractions are 2% and 8%, respectively. It is obvious from Figs. 12 and 13 and the aforementioned that lower Nd doping levels have smaller associated heat fractions and that the difference in heat fractions between no-stimulated-emission and strong saturation cases decreases with Nd doping concentration.

Finally, it is apparent from the results presented in this paper that measured values of heat fraction can be accounted for by quantum defect, concentration quenching, fluorescence, stimulated-emission heating, and a nonunity pump quantum efficiency. While the measurements discussed in [7]–[9] were performed with incident diode-pumped intensities that were low and in a regime where upconversion would be absent, it is also apparent that the theory presented here can also account for experimental data in which high incident diode intensities were used, and for the measured drop in heat fraction under strongly saturating laser conditions. In [14], for example, the measured drop in heating for a 1.1 at.% Nd-doped Nd:YAG rod in the presence of strong stimulated emission was 10%–15%, which is close to that calculated in Fig. 12, where for $K = 0$ and 0.155 we found the drop to be 8% and 14%, respectively. Also in [14], it was assumed that the heat fraction is 0.31, a value that agreed well with interferometric measurements under nonlasing conditions. From Fig. 9, we would expect a heat fraction under nonlasing conditions of about 0.4. The experiment in [14] was performed under high-intensity pumping conditions where the maximum pump intensity was 0.5 kW/cm$^2$, whereas the heat fraction measurements of [7] used pump intensities of less than 10 W/cm$^2$ or 50 times less. If upconversion were significant in [14] we would expect a greater rather than a smaller heat fraction. It is thus not likely that upconversion represents a significant heat-generation mechanism in CW diode-pumped Nd:YAG lasers.

IV. CW $\chi$ VALUES

In pulsed pumping, it is common to define a $\chi_p$ value, or the ratio of the time-integrated heat deposited divided by the peak
 energy stored. An explicit expression for $\chi_p$ will be presented in Section V. For CW pumping, we can define an analogous $\chi$ value $\chi_{cw}$ formally equal to the ratio of the CW heat power density or fraction generated divided by the CW inversion power density or fraction under conditions of no stimulated emission, or as simply the ratio of the total heat fraction with $\xi = 0$ divided by the inversion fraction

$$\chi_{cw}(\rho, \xi = 0) = \frac{\eta_h(\rho)}{\eta(\rho)}, \quad (45)$$

For no upconversion and no stimulated emission, (45) can be shown to be equal to

$$\chi_{cw}(\rho) = \frac{\eta_e(\rho) S_m}{\eta(\rho)(S_m - \phi_T)} (F(\rho) + 1) + \frac{1}{1 + \phi_T/S_m} (F(\rho) + S_j/S_m). \quad (46)$$

In Fig. 14, we show the calculated ratio of $\chi_{cw}(\rho)$, which increases as a function of Nd doping, for the two cases $K = 0$ and $K = 0.155$. Because of the larger heat fractions associated with a nonunity pump efficiency, the $\chi_{cw}$ values are larger for $K = 0.155$ than for $K = 0$.

For no stimulated emission, and by using the fact that the sum of the total heating and fluorescence fractions is equal to unity, it is also possible to calculate $\eta_h(\rho)$ values from a knowledge of $\eta_e(\rho)$ values only. From (41) and (43), we can calculate the ratio $\delta$ of fluorescence fraction to inversion fraction, obtaining

$$\delta = \frac{\eta_m - S_j}{\eta_m - \phi_T} = 1.020. \quad (47)$$

The ratio of fluorescence/inversion fractions is thus a constant. By using the definition of $\chi_{cw}(\rho) = \eta_h(\rho)/\eta(\rho)$ and (47), we find that $\chi_{cw}$ can be expressed as

$$\chi_{cw}(\rho) = \delta \left[ \frac{\eta_h(\rho)}{1 - \eta_e(\rho)} \right]. \quad (48)$$

and $\chi_{cw}$ can then be calculated as a function of $\eta_h$ only.

V. PULSED $\chi$ VALUE

Since there is much interest in pulsed lasers, in this section we present the relationships needed to calculate pulsed $\chi_p$ values. Since traditionally $\chi_p$ is calculated as the ratio of the total integrated heat generated during a pump pulse divided by the peak energy stored, without stimulated emission present, it is relatively easy to use the formalism developed in Section IV and apply it to this problem in order to obtain an explicit expression for $\chi_p$. Upconversion is ignored, however, it can easily be added by extension of the results presented in this paper.

Equation (2) can be solved for a pump pulse that is flattop, that is $R_e$ has a constant amplitude for times $t \leq T$ where $T$ is the pump pulsewidth, and $R_e = 0$ for $t > T$. We also assume $\gamma = \xi = 0$. Then the stored energy density can be found to be equal to

$$E_s(\rho, t) = \frac{\eta_e(\rho) R_e hc (\phi_m - \phi_T)}{k_R (F(\rho) + 1)} \times \left(1 - \exp(-k_R (F(\rho) + 1)t)\right) \quad (49)$$

and is a maximum for $t = T$. Similarly, we can find the time-integrated heat density $H_s$ by evaluating the equation

$$H_s(\rho) = \int_0^T Q_{h,T}(\rho, t) dt + \int_T^\infty Q_{h,T}(\rho, t) dt. \quad (50)$$

The first integral is evaluated with $R_e$ constant and thus gives the heat density generated during the pump pulse, while for the second integral, $R_e = 0$, and we evaluate the heating in the absence of pumping. Equation (50) can be easily evaluated; the result is the following equation:

$$H_s(\rho) = (\phi_p - \phi_e(\rho) \phi_m) R_e hc T + \frac{\eta_e(\rho) \phi_m R_e hc}{k_R} \times \left[\frac{F(\rho) + S_j}{(F(\rho) + 1)^2}\right] \times (k_R (F(\rho) + 1) T - 2 \exp(-k_R (F(\rho) + 1) T) - \exp(-2k_R (F(\rho) + 1) T) - 1). \quad (51)$$

By dividing (49) evaluated at $t = T$ by (51), we arrive at the final expression for $\chi_p(\rho)$, defined in (52), shown at the bottom of the next page.

Another way to derive $\chi_p(\rho)$ is to use energy rather than power fractions. We express $\chi_p(\rho)$ as

$$\chi_p(\rho) = \frac{\frac{H_s(\rho)}{E_{ab}(\rho, t = T)}}{\frac{E_{ab}(\rho, t = T)}{E_{ab}}} = \frac{\eta_{h,p}(\rho)}{\eta_{e,p}(\rho)} \quad (53)$$

where $E_{ab} = R_e hc \phi_p T$ is the energy absorbed during the pump pulse and $\eta_{h,p}(\rho)$ and $\eta_{e,p}(\rho)$ are the pulsed heat and
stored energy fractions, respectively. Calculation shows that
\( \eta_{p,p'}(\rho) \) and \( \eta_{q,p'}(\rho) \) are given by
\[
\eta_{p,p'}(\rho) = \left( 1 - \frac{\eta_k(p_m)}{\eta_k(p_p)} \right) + \frac{\eta_k(p)}{k_R(F(\rho) + 1)} \left( \frac{\rho_m}{\rho_p} \right) \left[ \frac{F(p) + S_T}{F(\rho) + 1} \right] \times \left[ k_R(F(\rho) + 1)T \right]
- \exp(-k_R(F(\rho) + 1)T) + 2 \exp(-k_R(F(\rho) + 1)T) \nonumber
\]
\[
+ \exp(-2k_R(F(\rho) + 1)T) - \exp(-k_R(F(\rho) + 1)T) - 1 \right] \right)
\]
(54)
and
\[
\eta_{q,p'}(\rho) = \frac{\eta_k(p)}{k_R(F(\rho) + 1)} \left( \frac{\rho_m}{\rho_p} - \frac{\rho_T}{\rho_p} \right) \cdot \left( 1 - \exp(-k_R(F(\rho) + 1)T) \right)
\]
(55)
Finally, we can then express \( \chi_p(\rho) \) in the following form:
\[
\chi_p(\rho) = \left[ \frac{\rho_p}{\rho_m - \rho_T} \right] \frac{\eta_{p,p'}(\rho)}{\eta_{q,p'}(\rho)} \frac{k_R(F(\rho) + 1)}{\eta_k(p)(1 - \exp(-k_R(F(\rho) + 1)T))}
\]
(56)
This equation is identical to that used by Fan [7] except that
(56) uses the pulsed heat fraction and [7] used a CW heat
fraction value.
In Fig. 15, we show calculated \( \chi_p(\rho) \) curves for pulsewidths
\( T \) of 100, 200, 300, 400, and 500 \( \mu s \), and for \( K = 0 \). The
\( \chi_p \) values increase with Nd concentration due to quenching
as well as with pulsewidth; longer pulsewidths linearly in-
crease the heat deposition energy but stored inversion energy
increases less than linearly with that parameter.
Fig. 16 shows calculated values of \( \chi_p(\rho) \) for \( K = 0 \) and
0.155 and a fixed pulsewidth of 200 \( \mu s \). At 1.04 at.% Nd
concentration, \( \chi_p \) takes the values 0.38 and 0.75 for \( K = 0 \)
and 0.155, respectively. Fan [7] calculated a \( \chi_p \) value of 0.9
for 1.04 at.% Nd doping and \( K = 0.10 \); the cause of this
discrepancy is the use of the CW rather than a pulsed value
for \( \eta_k \). If we substitute the CW rather than the pulsed \( \eta_k \) value
in (56) for \( K = 0.155 \), the \( \chi_p \) value then becomes equal to
0.90 at 1.04 at.% Nd doping, or close to the value derived by
In Fig. 16, we also show a data point taken from [10] where
\( \chi_p \) was measured to be 1.1 for 1.1 at.% Nd doping and a
200-\( \mu s \) pump pulsewidth. The value of the Nd:YAG effective
stimulated-emission cross section used to calculate \( \chi_p \) in [10]
was \( \sigma_e = 3.2 \times 10^{-10} \text{cm}^2 \) or the same value found by
Birnbaum et al. [30]. There are, however, a number of mea-
surements of \( \sigma_e \) reported in the literature [20], [24], [30]–[32].
We have calculated the average value \( (3.14 \times 10^{-10} \text{cm}^2) \)
and standard deviation \( (0.54 \times 10^{-10} \text{cm}^2) \) associated with five
measurements of \( \sigma_e \) [20], [24], [30]–[32] and find that when
the data in [10] is corrected the average \( \chi_p \) value is 1.08
while the minimum and maximum values are 0.89 and 1.26,
respectively. The lower value is close to that of Fan [7]; the
entire range of values are, however, larger than those we have
calculated, as shown previously in Fig. 16.

\[
\chi_p(\rho) = \frac{(\rho_p - \eta_k(p_m))k_R(F(\rho) + 1)T}{\eta_k(p)(\rho_m - \rho_T)(1 - \exp(-k_R(F(\rho) + 1)T)) + \frac{\rho_m}{\rho_p}} \left[ \frac{F(p) + S_T}{F(\rho) + 1} \right] \times \left[ k_R(F(\rho) + 1)T + 2 \exp(-k_R(F(\rho) + 1)T) - \exp(-2k_R(F(\rho) + 1)T) - 1 \right]
\]
(52)
It is not possible, in the absence of upconversion measurements in Nd:YAG, to say whether the difference might be attributed to that process. Experimental error and uncertainty in the value of the stimulated-emission cross section are also important factors to consider. Clearly, however, pulsed pumping results in larger inversion densities than those encountered in CW lasers and if upconversion becomes significant, could explain the anomalously large values for $\chi_p$ [10].

VI. CONCLUSION

In this paper, we have presented a comprehensive theory that allows the laser designer to explicitly calculate the CW heat, fluorescence, and stimulated-emission power densities and fractions for the well-known material Nd:YAG, as a function of Nd concentration and with or without stimulated emission present. We have shown that multophonon heating, concentration quenching, and a nonunity pump quantum efficiency can account for recently measured diode-pumped heat fractions, and that for CW Nd:YAG lasers upconversion is not an important heat-generating process. We also showed that for Nd dopings $>0.6$ at.%, the total heat fraction is less in the presence of stimulated emission. We have defined both CW and pulsed $\chi$ values and presented explicit expressions for their evaluation. Calculated pulsed $\chi$ values are not in agreement with measurement, a situation that can arise from measurement inaccuracies or the use of an inaccurate value of the stimulated-emission cross section. Upconversion, which does not appear to be important in CW Nd:YAG lasers, cannot at present be ruled out as a possible heat-generating mechanism in pulsed Nd:YAG devices where the inversion densities are significantly higher than those encountered in CW lasers. The present lack of information on upconversion in Nd:YAG makes it difficult to evaluate its importance for pulsed Nd:YAG lasers, however, we have included the effect in the model presented in this paper for future evaluation.

REFERENCES

[18] Data provided by Union-Carbide Chemicals and Plastics Company, Inc., Washougal, WA.
David C. Brown (M’87) received the Ph.D. degree in physics from Syracuse University, Syracuse, NY, in 1973 for research on He-Cd metal vapor lasers.

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